(1)

(a) $\frac{d}{dx}\cos(5x) = -\sin(5x)\cdot(5x)' = -5\sin(5x)$

(b) $\frac{d}{dx}\sin^2(x) = \frac{d}{dx}\left(\sin(x)\right)^2 = 2\sin(x)\cdot\frac{d}{dx}\sin(x)$

$= 2\sin(x)\cos(x)$

(c) $\frac{d}{dx} 4xe^{x^2+2} = (4x)'e^{x^2+2} + \left(e^{x^2+2}\right)'(4x)$

$(uv)' = u'v + v'u$

$= 4e^{x^2+2} + e^{x^2+2}\cdot(2x)(4x)$

$= (8x^2+4)e^{x^2+2}$

(d) $\frac{d}{dx}(x^2+x)^3\tan(2x) = 3(x^2+x)^2\cdot(2x+1)\tan(2x)$

$+ (x^2+x)^3\cdot\sec^2(2x)\cdot 2$

$= (x^2+x)^2(6x+3)\tan(2x)$

$+ 2(x^2+x)^3\sec(2x)$

(e) $\frac{d}{dx}\frac{x^3-2x^2}{\sin(x)} = \frac{(3x^2-4x)\sin(x) - \cos(x)\cdot(x^3-2x^2)}{\sin^2(x)}$

$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

②

(a) $\int \cos(10x)\,dx = \int \frac{1}{10}\cos(u)\,du = \frac{1}{10}\sin(u) + C$

$= \frac{1}{10}\sin(10x) + C$

$u = 10x$
$du = 10\,dx$
$\frac{1}{10}du = dx$

(b) $\int xe^{2x}\,dx = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\,dx$

$u = x \quad du = dx$
$dv = e^{2x} \quad v = \frac{1}{2}e^{2x}$
$\int u\,dv = uv - \int v\,du$

$= \frac{1}{2}xe^{2x} - \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

(c) $\int x^2\sin(2x)\,dx = -\frac{1}{2}x^2\cos(2x) + \int x\cos(2x)\,dx$

$u = x^2 \quad du = 2x\,dx$
$dv = \sin(2x) \quad v = -\frac{1}{2}\cos(2x)$
$\int u\,dv = uv - \int v\,du$

$= -\frac{1}{2}x^2\cos(2x) + \left[\frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)\,dx\right]$

$u = x \quad du = dx$
$dv = \cos(2x) \quad v = \frac{1}{2}\sin(2x)$
$\int u\,dv = uv - \int v\,du$

$= -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C$

(d) $\int \sin^2(x)\,dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right) dx$

$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$= \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$

$= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

(e) $\int \frac{\ln(x)}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C$

$u = \ln(x)$
$du = \frac{1}{x}\,dx$

$= \frac{1}{2}(\ln(x))^2 + C$

(f) $\int \frac{1}{x(\ln(x))^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-2+1}}{-2+1} + C$

$u = \ln(x)$
$du = \frac{1}{x}\,dx$

$= \frac{-1}{u} + C$

$= -\frac{1}{\ln(x)} + C$

(g) $\int \sin(x)\cos(x)\,dx = \int u\,du = \frac{1}{2}u^2 + C$

$u = \sin(x)$
$du = \cos(x)\,dx$

$= \frac{1}{2}\sin^2(x) + C$

(h) $\int \frac{e^x}{1+e^x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$

$= \ln|1+e^x| + C$

$= \ln(1+e^x) + C$

$u = 1+e^x$
$du = e^x\,dx$

since $1+e^x \geq 0$ we know $|1+e^x| = 1+e^x$

(i) $\int \frac{1}{1+e^x}\,dx = \int \frac{1+e^x-e^x}{1+e^x}\,dx = \int \frac{1+e^x}{1+e^x}\,dx - \int \frac{e^x}{1+e^x}\,dx$

$= \int 1\,dx - \int \frac{e^x}{1+e^x}\,dx$

$= x - \ln(1+e^x) + C$

trick:
$1 = 1+e^x - e^x$

see previous problem

(j) $\int e^{2x}\sin(e^x)\,dx = \int e^x \cdot e^x \cdot \sin(e^x)\,dx$

$= \int t\sin(t)\,dt = -t\cos(t) + \int \cos(t)$

$= -t\cos(t) + \sin(t) + C$

$= -e^x\cos(e^x) + \sin(e^x) + C$

$t = e^x$
$dt = e^x\,dx$

$u = t \quad du = dt$
$dv = \sin(t) \quad v = -\cos(t)$
$\int u\,dv = uv - \int v\,du$